

Why you can't measure zero

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Introduction

This document is provided to present some additional discussion on the subject of measurement criteria based on non-detectability. In particular, not detectable does not mean zero radioactivity concentration. To understand this, we need to examine the concept of minimum detectable concentration. This involves the Data Quality Objectives process and limiting decision error rates.

Step 1. Problem Statement

To determine if the material that is being measured contains radioactivity.

Step 2. Identify the Decision

To determine if the material must be treated as, or disposed of, as radioactive.

Step 3. Identify Inputs to the Decision

What level of radioactivity concentration in the material is unacceptable.

Step 4. Define the Study Boundaries

How much material is to be measured, what instrumentation/analysis is available, how much time and resources are available for the measurements.

Step 5. Develop a Decision Rule

Develop a rule so that when the measurement is made, I know what to do with the material. When the material is processed and inserted into an instrument, the measurement is made and the instrument output is a result that is a number.

The decision rule is to take that result number and compare it to a pre-determined number called the critical level. If the result is greater than the critical level, the decision is made to treat the material as radioactive.

(Usually, the instrument output and the critical level must be converted to a concentration value using a calibration factor. For simplicity, this example will assume that measurement instrument reads out

directly in concentration so that the calibration factor is one.)

Step 6. Specify Limits on Decision Errors

Decision errors?

The material either contains radioactivity or it does not. That is the truth. Unfortunately, we can never know the truth. We can only know the result of the measurement and then base our decision on it. There are four possibilities:

1. The material does not contain radioactivity, and after making the measurement we get a number below the critical value and so decide that it does not contain radioactivity.
2. The material does contain radioactivity, and after making the measurement we get a number above the critical value and so decide that it does contain radioactivity.
3. The material does not contain radioactivity, and after making the measurement we get a number above the critical value and so decide that it does contain radioactivity. This would be a decision error.
4. The material does contain radioactivity, and after making the measurement we get a number below the critical value and so decide that it does not contain radioactivity. This would also be a decision error.

Note that we can't ever know if we've made a decision error, we only know the result of the measurement. Measurements are not perfect and people make mistakes. Decision errors are unavoidable. However, recognizing that decision errors exist allows us to control their severity.

Several steps are necessary in order to create the framework for controlling decision errors:

Establish the Concentration Range of Interest

In step three of the DQO process we determined a level of radioactivity concentration in the material that is unacceptable. This is also sometimes called an *action level* (such as the derived concentration guideline level, a regulatory limit) that should not be exceeded. If the project planning team wants a method to measure sample concentrations around this level, they would not select one that worked at concentrations at 10 to 100 times the action level, nor would they select one that worked from zero to half the action level. They would want a method that worked well around the action level—perhaps from 0.1 to 10 times the action level, or from one-half to two times the action level. For the purpose of the example in this attachment, the action level is 1.0 and the project planning team selected a region of interest that is zero to twice the action level (0-2), as shown on the x-axis in Figure 1.

The first thing to notice is that figure 1 ranges from -1 to 2 and not 0 to 2. Why is this?

If I place a container of nothing in the instrument, the true concentration is zero. The instrument will produce a reading that is a number, and not necessarily the same exact number each time. If the zero point is set at the the average instrument reading when nothing is being measured, then the distribution of many measurements of nothing will look like Figure 2. The spread in these measurement results is characterized by the standard deviation of this distribution. In Figure 2, the standard deviation is 0.2. For the problem to be actually addressed, the standard deviation may be larger or smaller than this, but it will not be zero. There is always some variability in measurements, and this will always cause some uncertainty about whether or not the decisions based on these measurements are correct.

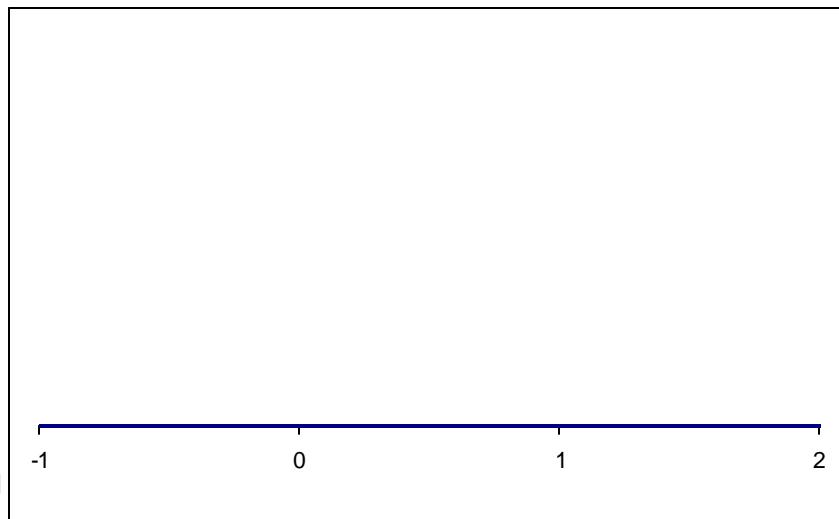


Figure 1

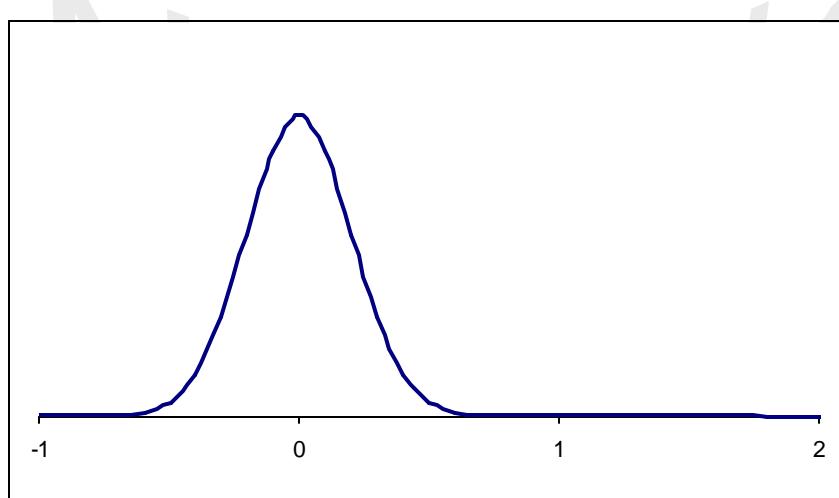


Figure 2

Consider a possible decision rule: Decide that there is radioactivity in the sample if the measurement result is greater than zero. (This means that the “critical value” is zero).

From figure 2 we can see that if I make the critical value for my decision equal to zero, I will decide that there is radioactivity in the sample about half the time, even when I am measuring nothing! Also notice that unless the instrument reading is negative, I cannot decide that there is no radioactivity in the sample. There is nothing contradictory about this. Remember, the zero point on the x-axis was chosen simply to be the spot where the average measurement of nothing fell. About half the time a measurement of nothing will be larger, and about half the time it will be smaller. This does not imply anything about concentrations being negative. It is about the variability of measurement readings, not the true concentration.

I might not be too happy about a decision rule that will lead me to the wrong conclusion half of the time. How can I improve this? Notice that if I make the critical value larger, I will conclude that there is radioactivity when there is nothing less often. If the critical value for the example is picked to be 0.2, I will only decide that there is radioactivity in the sample when the measurement result is greater than 0.2. From the example in Figure 3, I can estimate that this will happen about 15% of the time.

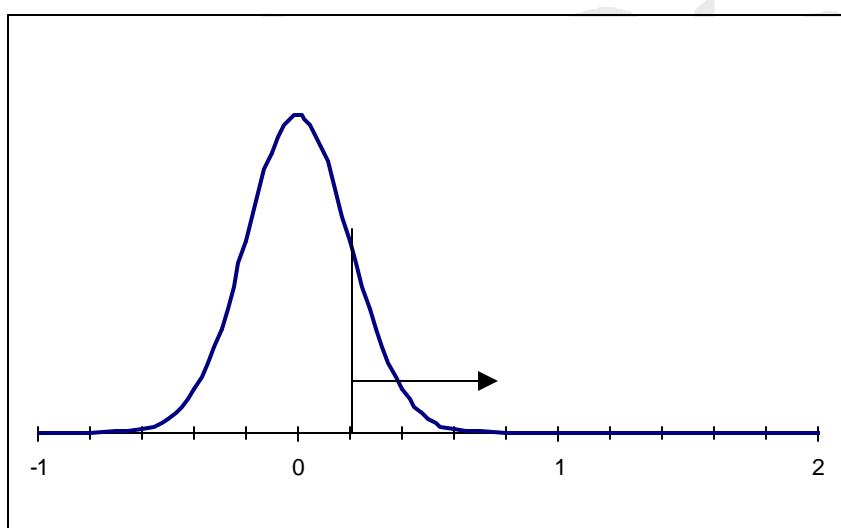


Figure 3

By making the critical value larger and larger, I can reduce the probability that I will say there is radioactivity when there is not to virtually zero. Great! Why not!

Well, unfortunately, there is a downside to doing that. To see that, we only have to turn the tables. Suppose instead of nothing, that there is a concentration of 0.2 in the sample. If I measure a sample

with this concentration many times, the distribution of results might look like Figure 4.

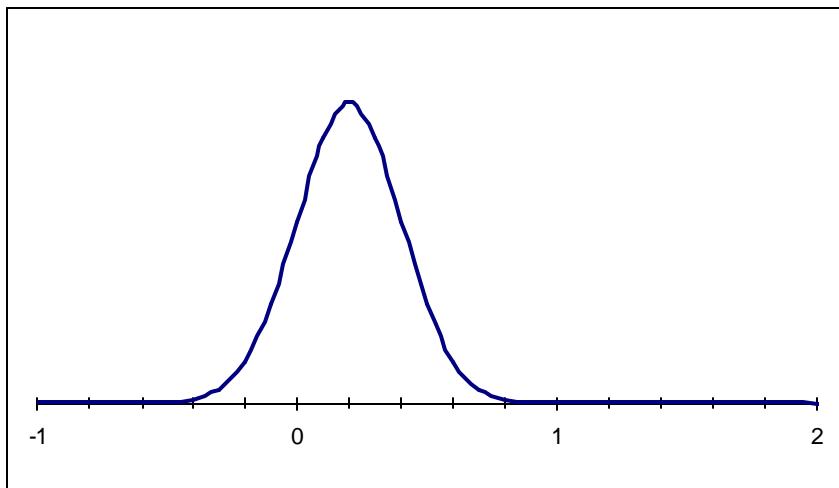


Figure 4

Notice that with a critical level of 0.2, we will only decide that there is radioactivity in this sample about half the time. Even when the critical level were zero, we would only decide that there is radioactivity in the sample about 85% of the time.

As we saw above, there are two types of decision errors that can be made: we can decide there is radioactivity when there is not, or we decide there is no radioactivity when there is. What the figures show is that by making the critical value for my decision rule bigger, I can reduce the chances of making first kind of decision error, but doing so will increase the chance of making the second kind of decision error. Making the critical value for my decision rule smaller will reduce chances of the second kind of decision error, but will increase the chance of the first kind of decision error.

There is no free lunch.

In this example we used a measurement variability (standard deviation) of 0.2. What if the variability is larger or smaller?

By looking at the figures we can conclude that *no matter what the variability actually is:*

- a) If a critical value of zero is used, I will conclude that there is radioactivity in a sample that actually contains nothing about half the time.
- b) If I pick a critical level equal to the standard deviation, I will conclude that there is radioactivity in a sample that actually contains nothing about 15% of the time.

(A slight modification of the figures would show that if the critical level equal to two times the standard

deviation, one will conclude that there is radioactivity in a sample that actually contains nothing about 2.5% of the time.)

- c) If a critical value of zero is used, I will conclude that there is *no* radioactivity in a sample that actually contains a concentration that is numerically equal to the standard deviation about 15% of the time. (A slight modification of the figures would show that if the critical level were equal to zero, one will conclude that there is *no* radioactivity in a sample that actually contains a concentration equal to twice the standard deviation about 2.5% of the time.)
- d) If I pick a critical level equal to the standard deviation, I will conclude that there is *no* radioactivity in a sample that actually contains a concentration numerically equal to the standard deviation about half the time.

The key is to notice that it is not the numerical value of the variability alone nor the numerical value of the concentration alone, that determines the probability of a decision error. It is the ratio of the concentration to the standard deviation that is important. In essence, the standard deviation determines the *scale* of the x-axis (concentration-axis) for this problem. Background determines where the zero point of the concentration axis.

The minimum detectable concentration (MDC) for a measurement process has been defined as the concentration which the sample must contain so that the probability of both kinds of decision errors are limited to 5%. This means that if a sample containing a concentration equal to the MDC is measured, about 95% of the time the measurement result will lead to the decision that the sample contains radioactivity. This is shown in Figure 5.

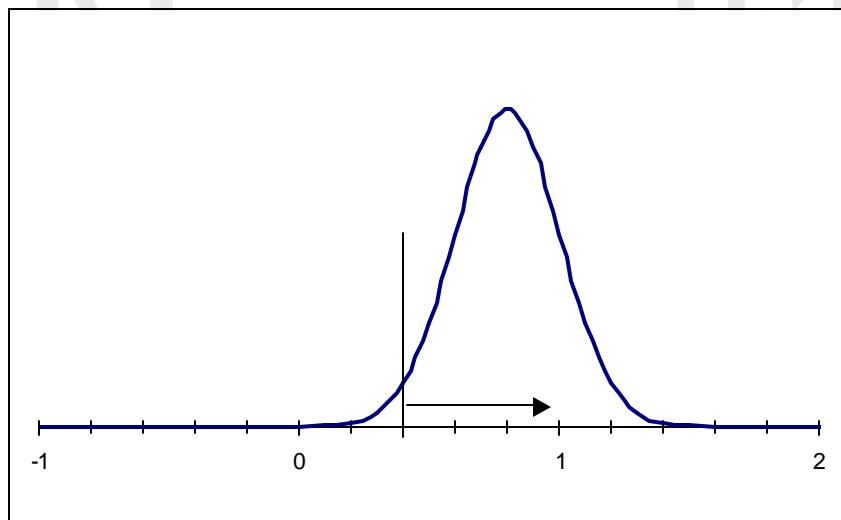


Figure 5

However, if a sample containing nothing is measured, the probability that the measurement result will lead to the decision that the sample contains radioactivity will be only about 5%. This is shown in Figure 6.

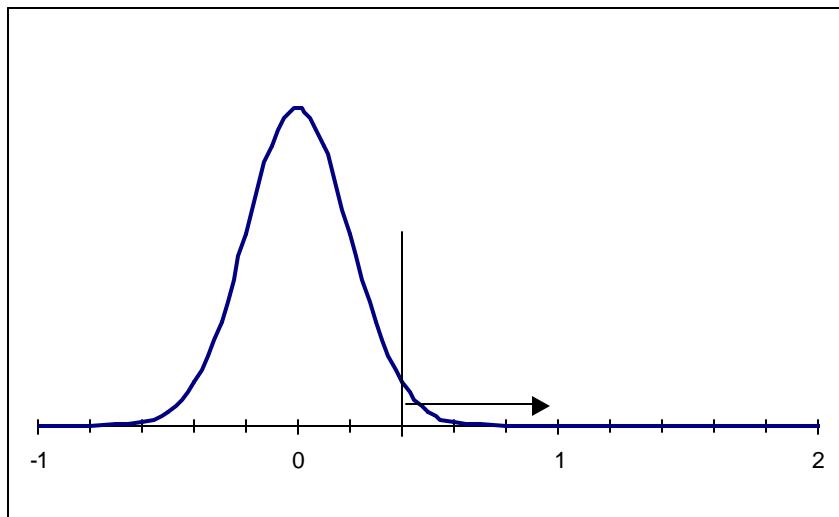


Figure 6

Depending on the specific details of the measurement process, the MDC is usually about four times the measurement variability when measuring nothing. The critical value being used, so that the error rates for both kinds of decision errors is limited to 5%, is about twice measurement variability when measuring nothing.

Estimate the Measurement Variability when Measuring Nothing

The measurement variability when measuring nothing is thus a key parameter for planning. The best way to get a handle on this is by making many measurements of nothing and computing the standard deviation of the measurement results.

What can be concluded about our ability to measure zero?

Bottom line: We can only reliably tell the difference between zero and the MDC.

If there is radioactivity present but at a concentration less than the MDC, I may be able to find it, but less than 95% of the time.

If the true concentration is at half the MDC (right at the critical value), half the time I will decide there is radioactivity, and half the time I will not.

The only way to do better is to reduce the measurement variability. This can usually only be done by either taking more measurements or by using an instrument or measurement process that has less variability when measuring nothing.

So what does it mean if I tell you that I measured a sample and decided there was no radioactivity.
(This is another way of saying that no radioactivity was detected.)

By itself, such a statement means nothing, and has no value unless I told you *the level of radioactivity that I could detect if it were there*, i.e. the MDC.

Similarly, to specify that no radioactivity be detectable in a sample, I must say how hard one must look. That is, I must specify an MDC, which in turn implies a certain limit on the variability of the measurement procedure.

In either case, I can never measure zero. I can only decide from measurement, with a prescribed limit on the probability of being wrong, that if enough radioactivity were there I would have found it. If I don't find it, it doesn't mean nothing is there. It only means that whatever might be there is unlikely to be more than the MDC.

Conclusion: I must determine an action level and make sure that my MDC is below it. Only then can I be sure that I can detect radioactivity concentrations that have an impact. Otherwise my measurements are a waste of money.

Conversely, specifying a measurement process implies an action level (level of concern) that is at or above the MDC.