

AUTOMATION OF MARKOV THEOREM FOR SOLUTION OF MULTISTATE SYSTEM PROBABILITIES

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Abstract

The objective of this paper is to utilize the Markov theory to compute time-dependent multistate system probabilities, and to demonstrate the method, using a diesel generator as a test case. To perform this analysis, a computer program (MARKOV-2) was developed to utilize Markov equations and calculate the probability of each system state as a function of time. Markov theory defines the relationship between the different states through the transition frequencies between those states, using a series of simultaneous linear differential equations. The MARKOV-2 computer program solves these equations using both implicit and explicit methods. These methods are applied in this paper to compute multistate diesel generator availability.

Introduction

Most system reliability analyses involve binary states (success/failure) and assume time independent behavior. These situations are analyzed with relatively simple models. In contrast, the analysis of reliability and availability of systems with multistates (e.g., failure, repair, operation, etc.) and time dependent behavior can become quite complex. To analyze these problems, it is necessary to define conditional probabilities $P(t)$ for each state. The probabilities for each state are a function of time and eventually reach equilibrium as time progresses. The Markov theory describes the equations relating different system states to each other. This process is accomplished through a series of simultaneous linear differential equations. These equations are solved either through an explicit method of estimating the final probabilities, or through an implicit method, in which the matrix of the coefficients are set up, and Gaussian elimination and back-substitution are used to calculate the state probabilities. MARKOV-2 is a microcomputer-based program that automates the Markov equations for multistate systems requiring state probability calculations.

Markov Theory

A system that consists of several states in addition to its success and failure state (e.g., standby, repair, test, etc.) can be in any one of these states at a specific time. This sort of model is generally called a Markov chain model or discrete-state, continuous-time model. One of the most important features of any Markov model is that the transition probability from state (i) to state (j) depends only on states (i) and (j) and is completely independent of all earlier states. The Markov theory (Ref. 1) uses the transition frequencies (λ) between the states to

calculate the conditional probabilities at time step dt around t . Eq. (1) shows the general form of the Markov equation.

For $i = 1$ to N

$$\frac{dP(s_i, t)}{dt} = \sum_{j=1}^N [\lambda_{sj \rightarrow si} * P(s_j, t)] - \sum_{j=1}^N [\lambda_{si \rightarrow sj} * P(s_i, t)] \quad (1)$$

Where:

$P(s_i, t)$ = Probability of being in state s_i at time t

$\lambda_{si \rightarrow sj}$ = Transition frequency of state s_i to state s_j

$P(s_j, t)$ = Probability of being in state s_j at time t

$\lambda_{sj \rightarrow si}$ = Transition frequency of state s_j to state s_i

Eq. (1) is in the form of simultaneous linear differential equations. The solution to these equations can be either explicit or implicit. These two methods are discussed below.

Explicit Method

The explicit method is the simplest approximation for the numerical solution of a differential equation. It assumes that the rate of change over Δt is only a function of initial values. The explicit method, approximates the values of "x" and "y" in the set of simultaneous linear differential equations as described in Eq. 2 through 7.

$$\frac{dx}{dt} = x + y \quad (2)$$

$$\frac{dy}{dt} = x - y \quad (3)$$

Therefore:

$$\frac{(x_2 - x_1)}{\Delta t} = x_1 + y_1 \quad (4)$$

$$\frac{(y_2 - y_1)}{\Delta t} = x_1 - y_1 \quad (5)$$

Therefore:

$$x_2 = x_1(1 + \Delta t) + y_1 \Delta t \quad (6)$$

$$y_2 = x_1 \Delta t + y_1(1 - \Delta t) \quad (7)$$

Where x_1 and y_1 are values of x and y at time t and x_2 and y_2 are values of x and y at time $t + \Delta t$.

Therefore, x_2 and y_2 are calculated as a function of time for given time steps Δt . Based on the solution shown in Eq. 6 and 7, the Markov equations can be written as in Eq. 8.

For $i = 1, \dots, N$

$$P(s_i, t + \Delta t) = P(s_i, t) + \Delta t * \left[\sum_{j=1}^N \lambda_{j \rightarrow i} * P(s_j, t) \right] - \Delta t * \left[\sum_{j=1}^N \lambda_{i \rightarrow j} * P(s_i, t) \right] \quad (8)$$

$P(s_i, t + \Delta t)$ is then calculated for each state. Note that when the value of Δt is large, the accuracy of the calculations is reduced considerably and will even become unstable resulting in negative probabilities. In the sample problem used to test MARKOV-2, when Δt was set to 4 h or higher, the results were not as accurate as for time steps less than 4 h.

Implicit Method

A better approximation of the system state probabilities can be derived by the implicit method. This method approximates the rate of change of the variable over Δt , as a function of both the initial value and final value of the variable. In this method, the solution for the x, y equations (Eq. 2,3) becomes:

$$\frac{(x_2 - x_1)}{\Delta t} = \frac{(x_1 + x_2)}{2} + \frac{(y_1 + y_2)}{2} \quad (9)$$

$$\frac{(y_2 - y_1)}{\Delta t} = \frac{(x_1 + x_2)}{2} - \frac{(y_1 + y_2)}{2} \quad (10)$$

Therefore:

$$x_2 * \left(\frac{1 - \Delta t}{2} \right) - y_2 * \left(\frac{\Delta t}{2} \right) = x_1 * \left(\frac{1 + \Delta t}{2} \right) + y_1 * \left(\frac{\Delta t}{2} \right) \quad (11)$$

$$x_2 * \left(\frac{-\Delta t}{2} \right) + y_2 * \left(\frac{1 + \Delta t}{2} \right) = x_1 * \left(\frac{\Delta t}{2} \right) + y_1 * \left(\frac{1 - \Delta t}{2} \right) \quad (12)$$

A matrix can be formed using the coefficients of x_2, y_2 and the right side of Eq. 11 and 12, in order to calculate the values for "x" and "y." Applying this methodology to the Markov equation (Eq. 1) will result in:

For $i = 1, \dots, N$

$$\left[1 + \left(\frac{\Delta t}{2} \right) * \sum_{j=1}^N \lambda_{i \rightarrow j} \right] * P(s_i, t + \Delta t) - \left(\frac{\Delta t}{2} \right) * \sum_{j=1}^N \lambda_{j \rightarrow i} * P(s_j, t + \Delta t) = \left[1 - \left(\frac{\Delta t}{2} \right) * \sum_{j=1}^N \lambda_{i \rightarrow j} \right] * P(s_i, t) + \left(\frac{\Delta t}{2} \right) * \sum_{j=1}^N \lambda_{j \rightarrow i} * P(s_j, t) \quad (13)$$

Eq. 13 is used in MARKOV-2 to calculate the values for $P(s_i, t + \Delta t)$ using Gaussian elimination and back-substitution

(Ref. 2). The results of the implicit approach are more stable than the explicit method, especially for larger time steps. For time steps, smaller than the characteristic times of the equations (3 h or less in the case of the sample problem used to test MARKOV-2), the two methods give identical results.

MARKOV-2 Computer Program

MARKOV-2 is written in BASIC language. The code is set up to analyze any system with multistates. Figure 1 shows the program flow chart. As shown, the input includes the method of calculation (implicit or explicit), and the specific data concerning system states and operation. The output includes a brief summary concerning some of the input parameters such as the method of calculation, number of states, time step and duration, and transition frequencies. The output also shows a listing of the probability of each state at every time step.

Application of MARKOV-2 to Emergency Diesel Generator

MARKOV-2 was tested using an emergency diesel generator (DG) with six states. The possible states of the DG are:

1. Standby
2. Operation
3. Test
4. Preventive maintenance (PM)
5. Failure
6. Repair

The transition matrix among these six states is shown in Table 1. As shown in Figure 2, each state is related to the other

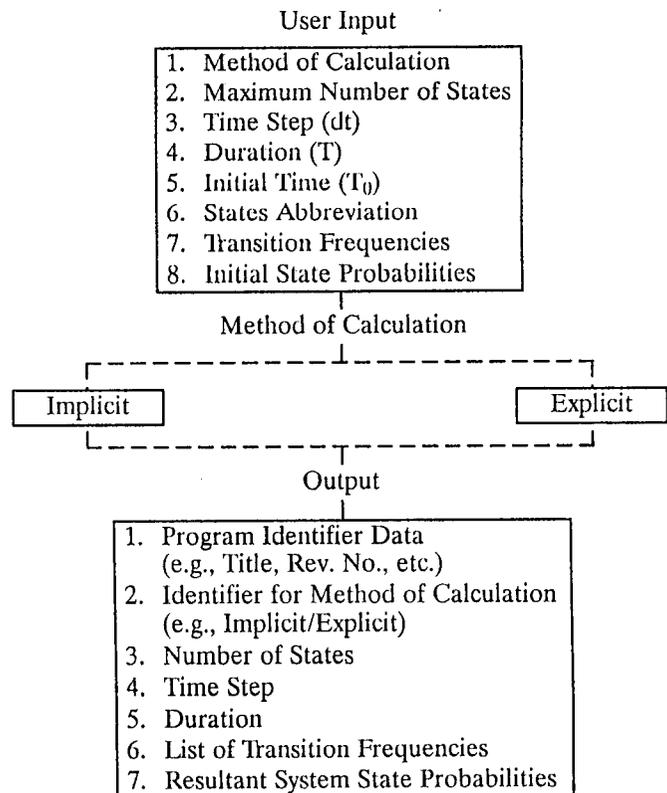


Figure 1. Program Flow Path

Table 1. Transition Matrix for DG Markov States

To From	Standby	Operat- ing	Test- ing	PM	Failed	Repair
Standby	X	✓	✓	✓	✓	X
Operating	✓	X	X	X	✓	X
Testing	✓	✓	X	✓	✓	X
PM	✓	✓	✓	X	✓	✓
Failed	X	X	X	X	X	✓
Repair	✓	X	✓	X	✓	X

X = No transition
 ✓ = Transition

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by a transition frequency (λ). Note that specific allowable states and transitions will be different for different systems and missions (e.g., operating systems will not have standby states). The DG, however, is a safety system which includes standby as its normal state, and operation, test, PM, and repair as its intermediate states. The methodology for the calculation of DG transition frequencies consists of utilizing the relationship between each state with the other states (Appendix A). For example, as shown in Appendix A, to calculate the transition frequency from standby to operating mode, one has to utilize the frequency of loss of offsite power (LOOP), since the DG is expected to start and operate under this condition. The parameters used to calculate transition frequencies, their values and the source of data is shown in Table A.1. Note that in some cases, engineering estimates were used. Table A.2 summarizes the equations used in the calculation of transition frequencies.

Table A.3 includes the printout of the MARKOV-2 output using the implicit method. The output is based on 10-h time steps. Some of the input data such as number of DG states (six states), time step (10 h), duration (100 h) and transition frequencies among various DG states are also listed. Table A.3

also shows the calculated probabilities for each state as a function of time. It was assumed that the DG was in standby state at time zero ($T=0$) and therefore, its probability was 1.0. Meanwhile the probabilities of other states were zero at that time. With the increased time, each state probability reaches equilibrium and remains constant for the rest of the duration. To test the accuracy of these probabilities, they can be compared with "time-independent" hand calculations. The hand calculations approximate the probability of a certain state based on the frequency of occurrence and the duration of a specific state. For example, it is assumed that the frequency of LOOP is 0.1 per year. If we conservatively assume that DG is required to start and operate for 24 h following a LOOP, the probability of "operation" state can be approximated as follows:

$$\left(\frac{0.1 \text{ LOOP}}{\text{yr}}\right) * \left(\frac{\text{yr}}{8760 \text{ h}}\right) * (24 \text{ h of DG operation}) = 2.74\text{E}-04$$

Table A.3 shows an equilibrium value of 2.47E-04 for the operation state that is close to the hand calculation value. The rest of the state probabilities can also be hand calculated based on their frequency of occurrence and duration. This test shows that the "time-independent" calculations are relatively good approximations of the equilibrium values of the more rigorous Markov calculation.

Summary and Conclusions

A PC-based program has been developed that utilizes Markov theory to compute time-dependent multistate probabilities. Any number of states can be modeled, and the solution can be either explicit or implicit. The program has been demonstrated using a six-state diesel generator model. The methodology is useful when information on transient (nonequilibrium) state probabilities are required and can be used to optimize maintenance and testing schedules.

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Appendix A – Method of Calculation of $\lambda(I,J)$ for Emergency Diesel Generator

Emergency diesel generators (DGs) are assumed to be in one of the following six states during their lifetime:

1. Standby
2. Operation
3. Test
4. Preventive maintenance (PM)
5. Failure
6. Repair

The methodology for calculation of the transition frequencies among these six states is described below:

1. $\lambda_{(S \rightarrow O)}$

Frequency of DG demand = Frequency of loss of offsite power (LOOP) = 0.1/yr

$$\lambda_{S \rightarrow O} = \frac{0.1}{(8760 \text{ h})} = 1.14E - 05/\text{h}$$

2. $\lambda_{(S \rightarrow T)}$

Assuming testing is done on a DG once per month:

$$\lambda_{S \rightarrow T} = \frac{1}{(730 \text{ h})} = 1.37E - 03/\text{h}$$

3. $\lambda_{(S \rightarrow P)}$

Assume PM is done on a DG every other month (once every 60 d):

$$\lambda_{S \rightarrow P} = \frac{1}{(2 * 730 \text{ h})} = 6.85E - 04/\text{h}$$

4. $\lambda_{(S \rightarrow F)}$

Approach a. Based on the assumption made in case 2 above, testing is done on the DG every month.

$$\lambda_{S \rightarrow F} = \frac{2.0E - 02}{(730 \text{ h})} = 2.74E - 05/\text{h}$$

Approach b. According to IREP*, failure rate to run given start for a DG is 3.0E-03/h (mean). Assuming $\lambda_{S \rightarrow F}$ is lower by a factor of 10:

$$\lambda_{S \rightarrow F} = 3.0E - 04/\text{h} \text{ (more conservative)}$$

5. $\lambda_{(S \rightarrow R)}$

In the case of DG, this is not a meaningful transition.

$$\lambda_{S \rightarrow R} = 0$$

6. $\lambda_{(O \rightarrow S)}$

Assuming a DG operational duration of 24 h, the transition to standby will occur at the end of operation.

$$\lambda_{O \rightarrow S} = \frac{1}{(24 \text{ h})} = 4.2E - 02/\text{h}$$

7. $\lambda_{(O \rightarrow T)}$

In the case of DG, this is not a meaningful transition.

$$\lambda_{O \rightarrow T} = 0$$

8. $\lambda_{(O \rightarrow P)}$

In the case of DG, this is not a meaningful transition.

$$\lambda_{O \rightarrow P} = 0$$

9. $\lambda_{(O \rightarrow F)}$

Approach a. Assuming that $\lambda_{O \rightarrow F}$ is 10 times higher than $\lambda_{S \rightarrow F}$ (see 4a.):

$$\lambda_{O \rightarrow F} = 2.74E - 04/\text{h}$$

Approach b. As mentioned in case 4b, IREP set the failure rate to run given start for a DG to 3.0E-03/h (mean). Therefore:

$$\lambda_{O \rightarrow F} = 3.0E - 03/\text{h} \text{ (more conservative)}$$

10. $\lambda_{(O \rightarrow R)}$

In the case of DG, this is not a meaningful transition.

$$\lambda_{O \rightarrow R} = 0$$

11. $\lambda_{(T \rightarrow S)}$

It is assumed that testing is done on DG every month for 2 h. Assuming that 90% of the time, the test done on DG is successful and it is set to standby mode afterwards:

$$\lambda_{T \rightarrow S} = 0.9 * \frac{1}{(2 \text{ h})} = 0.45/\text{h}$$

*IREP = Interim Reliability Evaluation Program Procedures Guide

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12. $\lambda_{(Test \rightarrow Operation)}$

During DG test, the DG override capability allows automatic transformation of DG test mode to emergency mode upon LOOP. Conservatively assuming that 99% of the time this transformation will take place:

$$\lambda_{T \rightarrow O} = 0.99 * \frac{0.1}{(8760 \text{ h})} = 1.1E - 05/h$$

13. $\lambda_{(Test \rightarrow PM)}$

As in case 11, assuming 7% of DG tests need transition to PM state:

$$\lambda_{T \rightarrow P} = 0.07 * \frac{1}{(2 \text{ h})} = 3.5E - 02/h$$

14. $\lambda_{(Test \rightarrow Failure)}$

As in case 13, assuming 3% of DG tests fail:

$$\lambda_{T \rightarrow F} = 0.03 * \frac{1}{(2 \text{ h})} = 1.5E - 02/h$$

15. $\lambda_{(Test \rightarrow Repair)}$

In the case of DG, this is not a meaningful transition (it is usually, test \rightarrow failure \rightarrow repair).

$$\lambda_{T \rightarrow R} = 0$$

16. $\lambda_{(PM \rightarrow Standby)}$

Assume that PM is done on DG every other month for a period of two shifts (16 h). Also assuming that 50% of PM are routinely returned to standby on completion (without testing):

$$\lambda_{P \rightarrow S} = 0.5 * \frac{1}{(16 \text{ h})} = 3.1E - 02/h$$

17. $\lambda_{(PM \rightarrow Standby)}$

Although in most cases DG will be out of service (OOS) during PM, assuming 10% of PMs can return the DG on line upon LOOP:

$$\lambda_{P \rightarrow O} = 0.1 * \frac{0.1}{(8760 \text{ h})} = 1.1E - 06/h$$

18. $\lambda_{(PM \rightarrow Test)}$

Assuming approximately 43% of PM activities require testing prior to return to standby, therefore:

$$\lambda_{P \rightarrow T} = 0.43 * \frac{1}{(16 \text{ h})} = 2.72E - 02/h$$

19. $\lambda_{(PM \rightarrow Failure)}$

During PM it is assumed that there are no major problems with DG and the maintenance is only scheduled preventive. However, human error or procedure diversion/errors can cause system failure while performing PM. Assuming a total of 5% of PM activities will result in system failure:

$$\lambda_{P \rightarrow F} = 0.05 * \frac{1}{(16 \text{ h})} = 3.1E - 03/h$$

20. $\lambda_{(PM \rightarrow Repair)}$

Since during PM on DG, discovery of leaks in starting air, control air, lube oil, etc., is possible, work order can be written for corrective maintenance (CM) while performing PM. Assuming probability for failure potential is 2%:

$$\lambda_{P \rightarrow R} = 0.02 * \frac{1}{(16 \text{ h})} = 1.25E - 03/h$$

21. $\lambda_{(Failure \rightarrow Standby)}$

In the case of DG, this is not a meaningful transition:

$$\lambda_{F \rightarrow S} = 0$$

22. $\lambda_{(Failure \rightarrow Operation)}$

In the case of DG, this is not a meaningful transition:

$$\lambda_{F \rightarrow O} = 0$$

23. $\lambda_{(Failure \rightarrow Test)}$

In the case of DG, this is not a meaningful transition:

$$\lambda_{F \rightarrow T} = 0$$

24. $\lambda_{(Failure \rightarrow PM)}$

In the case of DG, this is not a meaningful transition:

$$\lambda_{F \rightarrow P} = 0$$

25. $\lambda_{(Failure \rightarrow Repair)}$

Since the DG is considered a safety system, as soon as a failure is detected, the plant goes into a LCO (assuming hot shutdown or higher mode of operation). Assuming a mean diagnosis time of 4 h:

$$\lambda_{F \rightarrow R} = 1/(4 \text{ h}) = 2.5E - 01/h$$

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26. $\lambda_{(\text{Repair} \rightarrow \text{Standby})}$

Assuming that the mean repair time is 8 h and 60% of repairs do not require testing:

$$\lambda_{R \rightarrow S} = 0.6 * \frac{1}{(8 \text{ h})} = 7.5E - 02/\text{h}$$

27. $\lambda_{(\text{Repair} \rightarrow \text{Operation})}$

In the case of DG, this is not a meaningful transition (if the DG is under repair, it implies that it is OOS).

$$\lambda_{R \rightarrow O} = 0$$

28. $\lambda_{(\text{Repair} \rightarrow \text{Test})}$

Assuming that the mean repair time is 8 h and 35% of repairs require testing:

$$\lambda_{R \rightarrow T} = 0.35 * \frac{1}{(8 \text{ h})} = 4.38E - 02/\text{h}$$

29. $\lambda_{(\text{Repair} \rightarrow \text{PM})}$

In the case of DG, this is not a meaningful transition:

$$\lambda_{R \rightarrow O} = 0$$

30. $\lambda_{(\text{Repair} \rightarrow \text{Failure})}$

Assuming 5% of repair is done incorrectly due to human errors or failure to follow procedures:

$$\lambda_{R \rightarrow F} = 0.05 * \frac{1}{(8 \text{ h})} = 6.25E - 03/\text{h}$$

References

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Table A.1. Parameters for Calculation of DG Transition Rates

Symbol	Name	Value	Source
λ_{LOOP}	Frequency of loss of offsite power	0.1/yr	PRA value for LOOP
T_{TI}	Test interval	730 h (1 month)	Estimate
T_{PI}	PM interval	1460 h (2 months)	Estimate
P_{FS}	Probability of DG failure to start (per demand)	2.0E-02/d	Estimate
λ_R	Failure to run given start	3.0E-03/h	IREP
T_{OD}	Operational duration	24 h	Estimate
m_{O-S}	Operational to standby failure rate multiplier	10	Estimate
T_{TD}	Test duration	2 h	Estimate
P_{TS}	Probability that test is successful	90%	Estimate
P_{T-O}	Probability that DG goes to emergency mode from test mode	99%	Estimate
P_{T-P}	Probability that DG does to PM mode from test mode	7%	Estimate
P_{T-F}	Probability that DG fails during test mode	3%	Estimate
P_{P-S}	Probability that DG returns to standby mode from PM mode	50%	Estimate
T_{PD}	PM duration	16 h	Estimate
P_{P-O}	Probability that DG is demanded upon completion of PM	10%	Estimate
P_{P-T}	Probability that DG is tested after PM	43%	Estimate
P_{P-F}	Probability that DG fails due to human error after PM	5%	Estimate
P_{P-R}	Probability that DG requires CM after performing PM	2%	Estimate
T_{MDT}	Mean diagnosis time	4 h	Estimate
T_{MRT}	Mean repair time	8 h	Estimate
P_{R-S}	Probability that DG goes to standby mode from repair mode with no testing	60%	Estimate
P_{R-T}	Probability that DG is tested after repair	35%	Estimate
P_{R-F}	Probability that DG fails after repair	5%	Estimate

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Table A.2. Transition Frequency Equations

1. $\lambda_{S \rightarrow O} = \lambda_{LOOP} * \frac{1}{8760 \text{ h}}$	16. $\lambda_{P \rightarrow S} = P_{P-S} * \frac{1}{T_{PD}}$
2. $\lambda_{S \rightarrow T} = \frac{1}{T_{TI}}$	17. $\lambda_{P \rightarrow O} = P_{P-O} * \lambda_{LOOP} * \frac{1}{8760 \text{ h}}$
3. $\lambda_{S \rightarrow P} = \frac{1}{T_{PI}}$	18. $\lambda_{P \rightarrow T} = P_{P-T} * \frac{1}{T_{PD}}$
4. $\lambda_{S \rightarrow F} = \frac{\lambda_R}{m_{OS}}$	19. $\lambda_{P \rightarrow F} = P_{P-F} * \frac{1}{T_{PD}}$
5. $\lambda_{S \rightarrow R} = 0$	20. $\lambda_{P \rightarrow R} = P_{P-R} * \frac{1}{T_{PD}}$
6. $\lambda_{O \rightarrow S} = \frac{1}{T_{OD}}$	21. $\lambda_{F \rightarrow S} = 0$
7. $\lambda_{O \rightarrow T} = 0$	22. $\lambda_{F \rightarrow O} = 0$
8. $\lambda_{O \rightarrow P} = 0$	23. $\lambda_{F \rightarrow T} = 0$
9. $\lambda_{O \rightarrow F} = \lambda_R$	24. $\lambda_{F \rightarrow P} = 0$
10. $\lambda_{O \rightarrow R} = 0$	25. $\lambda_{F \rightarrow R} = \frac{1}{T_{MDT}}$
11. $\lambda_{T \rightarrow S} = P_{TS} * \frac{1}{T_{TD}}$	26. $\lambda_{R \rightarrow S} = P_{R-S} * \frac{1}{T_{MRT}}$
12. $\lambda_{T \rightarrow O} = P_{T-O} * \lambda_{LOOP} * \frac{1}{8760 \text{ h}}$	27. $\lambda_{R \rightarrow O} = 0$
13. $\lambda_{T \rightarrow P} = P_{T-P} * \frac{1}{T_{TD}}$	28. $\lambda_{R \rightarrow T} = P_{R-T} * \frac{1}{T_{MRT}}$
14. $\lambda_{T \rightarrow F} = P_{T-F} * \frac{1}{T_{TD}}$	29. $\lambda_{R \rightarrow P} = 0$
15. $\lambda_{T \rightarrow R} = 0$	30. $\lambda_{R \rightarrow F} = P_{R-F} * \frac{1}{T_{MRT}}$

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Table A.3. MARKOV-2 Sample Output

System State Probability Calculations Using Implicit Approach						
Number of States = 6 Time Step (h) = 10 Duration (h) = 100						
List of Transition Frequencies (/h)						
From \ To	STDBY	OPER	TEST	PM	FAIL	REPR
STDBY	0.00E+00	1.14E-05	1.37E-03	6.85E-04	3.00E-04	0.00E+00
OPER	4.20E-02	0.00E+00	0.00E+00	0.00E+00	3.00E-03	0.00E+00
TEST	4.50E-01	1.10E-05	0.00E+00	3.50E-02	1.50E-02	0.00E+00
PM	3.10E-02	1.10E-06	2.70E-02	0.00E+00	3.10E-03	1.25E-03
FAIL	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.50E-01
REPR	7.50E-02	0.00E+00	4.38E-02	0.00E+00	6.25E-03	0.00E+00
Resultant System State Probabilities						
Time (h)	STDBY	OPER	TEST	PM	FAIL	REPR
	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)
0	1.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
10	9.87E-01	9.27E-05	4.19E-03	5.75E-03	1.52E-03	1.19E-03
20	9.84E-01	1.51E-04	2.88E-03	9.10E-03	1.54E-03	2.68E-03
30	9.81E-01	1.87E-04	3.74E-03	1.08E-02	1.58E-03	3.09E-03
40	9.80E-01	2.10E-04	3.50E-03	1.17E-02	1.62E-03	3.26E-03
50	9.79E-01	2.24E-04	3.67E-03	1.22E-02	1.62E-03	3.34E-03
60	9.79E-01	2.34E-04	3.63E-03	1.25E-02	1.63E-03	3.37E-03
70	9.78E-01	2.39E-04	3.67E-03	1.26E-02	1.64E-03	3.39E-03
80	9.78E-01	2.43E-04	3.66E-03	1.27E-02	1.64E-03	3.40E-03
90	9.78E-01	2.45E-04	3.67E-03	1.28E-02	1.64E-03	3.40E-03
100	9.78E-01	2.47E-04	3.67E-03	1.28E-02	1.64E-03	3.41E-03
110	9.78E-01	2.47E-04	3.67E-03	1.28E-02	1.64E-03	3.41E-03