BINOMIAL EXPANSION THEOREM FOR EVALUATING REDUNDANT SYSTEMS

| |
|---|
| J-out-of-K for success<br>$A = \frac{K!}{(K-J)!J!} (a)^J (1-a)^{K-J}$ |
| J-out-of-K for failure<br>$U = \frac{K!}{(K-J)!J!} (u)^J (1-u)^{K-J}$ |
| ≥J-out-of-K for success<br>$A = \sum_{H=J}^{K} \frac{K!}{(K-H)!H!} (a)^H (1-a)^{K-H}$ |
| ≥J-out-of-K for failure<br>$U = \sum_{H=J}^{K} \frac{K!}{(K-H)!H!} (u)^H (1-u)^{K-H}$ |

where:

a = component availability

u = 1 - a = component unavailability

A = redundant system availability

U = 1 - A = redundant system unavailability.

J-out-of-K for success

$$R = \frac{K!}{(K - J)!J!} (r)^J (1 - r)^{K-J}$$

J-out-of-K for failure

$$U = \frac{K!}{(K - J)!J!} (u)^J (1 - u)^{K-J}$$

≥J-out-of-K for success

$$R = \sum_{H=J}^{K} \frac{K!}{(K - H)!H!} (r)^H (1 - r)^{K-H}$$

≥J-out-of-K for failure

$$U = \sum_{H=J}^{K} \frac{K!}{(K - H)!H!} (u)^H (1 - u)^{K-H}$$

where:

$r$ = component reliability

$u = 1 - r$ = component unreliability

$R$ = redundant system reliability

$U = 1 - R$ = redundant system unreliability

**Binomial Expansion Theorem for Evaluating Redundant Systems**

## DIRECT QUANTIFICATION ALGORITHMS FOR FAILURE STATE DIAGRAMS

As explained in Section 4.1, expanding the shorthand form of the failure state diagrams to explicitly represent every component would show a very large number of minimal cut sets. We, however, want to be able to quantify these FSDs without explicitly writing down minimal cut sets. To do this, we first define the notation $(q)_K^J$.

This term represents the probability of failure of a subsystem, comprised of K replicate components whose individual unavailabilities are q, and which will fail if at least J-out-of-K components fail. The evaluation of this term when each component is independent is standard and utilizes the binomial expansion

$$(q)_K^J = \sum_{H=J}^{K} \frac{K!}{(K-H)!H!} (q)^H (1-q)^{K-H}$$

To facilitate using the statistical correlation method in these expressions, it is necessary to expand the polynomial $(1-q)^{K-H}$, again with the binomial expansion, such that $(q)_K^J$ can be expressed in terms of powers of q.

$$(1-q)^{K-H} = \sum_{I=0}^{K-H} \frac{(K-H)!}{(K-H-I)!I!} (q)^I (-1)^I$$

Substituting into $(q)_K^J$

$$(q)_K^J = \sum_{H=J}^{K} \left( \frac{K!}{(K-H)!H!} (q)^H \sum_{I=0}^{K-H} \frac{(K-H)!}{(K-H-I)!I!} (q)^I (-1)^I \right)$$

$$(q)_K^J = \sum_{H=J}^{K} \sum_{I=0}^{K-H} \frac{K!}{(K-H-I)!H!I!} (q)^{H+I} (-1)^I$$

Table 4.1 shows some common expressions for J-out-of-K logic when J and K small. When J and/or K are large, the expressions become too complex for d calculation but can nevertheless be evaluated on the computer using the ve double summation series.

TABLE 4.1

J-OUT-OF-K REPLICATE COMPONENTS

| Logic | Probability of System Failure |
|---|---|
| 1/1.F | q* |
| ≥1/2.F | $2q - q^2$ |
| 2/2.F | $q^2$ |
| ≥1/3.F | $3q - 3q^2 + q^3$ |
| ≥2/3.F | $3q^2 - 2q^3$ |
| 3/3.F | $q^3$ |
| ≥1/4.F | $4q - 6q^2 + 4q^3 - q^4$ |
| ≥2/4.F | $6q^2 - 8q^3 + 3q^4$ |
| ≥3/4.F | $4q^3 - 3q^4$ |
| 4/4.F | $q^4$ |
| . | |
| . | |
| . | |

*q = failure probability of single replicate component

F = represents failure logic