

## Lab O1: Radioactivity and Counting Statistics

### Radioactivity

Radioactivity usually results from nuclear reactions, that is, reactions which involve the breaking of nuclear bonds having energies of the order of  $10^6$  eV = 1MeV. The three most common types of radioactive emissions are:

alpha rays:  $\alpha$  rays or  $\alpha$  particles are high-energy helium nuclei (2 protons + 2 neutrons), which are spontaneously emitted during the decay of radioactive nuclei.  $\alpha$ -rays are extremely dangerous, but, fortunately, they have very little penetrating power. Several centimeters of air or a sheet of paper will stop  $\alpha$  rays. The primary danger of  $\alpha$  sources comes from inhalation or ingestion of trace sources — so... no smoking or eating allowed when handling  $\alpha$  sources.

beta rays:  $\beta$  rays are high energy electrons emitted by nuclear reactions.  $\beta$  rays are generally less dangerous than alpha rays but have greater penetrating power. A thick plate of Plexiglas or a thin sheet of metal are needed to stop  $\beta$ 's.

gamma rays:  $\gamma$  rays are high-energy photons, i.e. particles of electromagnetic radiation, like x-rays, but with higher energy, and more dangerous, with much greater penetrating power. x-rays in medical applications are typically 20 keV in energy; gamma rays from nuclear reactions have energies of MeV. Several inches of lead are needed to completely stop high-energy  $\gamma$ 's

We are continuously exposed to radioactivity from natural sources: mainly naturally-occurring radioactive nuclei in rocks and cosmic rays. Cosmic rays are extremely high-energy charged particles, mostly hydrogen and helium nuclei, of extra-galactic origin, which strike the earth from all directions. The earth's atmosphere protects us from direct exposure to cosmic rays, but when they strike the upper atmosphere they precipitate a cascade of nuclear reactions whose decay products reach the ground. Here in Boulder, at an altitude of one mile, the natural radioactivity from cosmic rays is about twice as great as at sea level. Long-lived radioactive elements, mainly uranium and thorium, have been in the earth since its creation 4.8 billion years ago. These massive nuclei are unstable and spontaneously decay by fission into lighter elements (called daughter products), some of which are also radioactive. Particularly dangerous is the daughter product strontium-90 which is chemically similar to calcium and, when eaten or inhaled, is retained by the body and concentrates in bone tissue.

One way to describe the strength of a radioactive source is to give its *activity*, which is the number of radioactive decay events per second. The **curie** (Ci) is a unit of activity equal to  $3.7 \times 10^{10}$  decays/sec; this is approximately the activity of 1 gram of radium. Generally speaking, radioactive samples with an activity of 1  $\mu$ Ci or less are fairly safe to handle and can be purchased without a license from the NRC (Nuclear Regulatory Commission). Sources of 1 mCi or greater can be quite dangerous. Their use is carefully regulated and they must be handled with the utmost respect. Another unit of radiation is the **rad**, (short for radiation absorbed dose), which describes the *dose* which an exposed person receives. A rad is the amount of radiation which deposits 0.01 J of energy into 1 kg of absorbing material.

Neither the curie nor the rad can be used to adequately describe the biological damage due to radiation, because such damage depends strongly on the type of the radiation.  $\alpha$  is the

most dangerous, followed by  $\beta$ , then  $\gamma$ . A 1 rad dose of  $\alpha$  radiation does about 20 times more damage to cell tissue than a 1 rad dose of  $\gamma$  radiation. The **rem** is a unit which takes into account both the dose in rad and the type of radiation

dose in rem = dose in rad  $\times$  RBE factor (relative biological effectiveness)

RBE = 1 for  $\gamma$ ,  $\approx 1.6$  for  $\beta$ , and 20 for  $\alpha$ .

source/situation	dose	effect
neutron bomb blast	>100,000 rem	immediate death
Chernobyl firefighter	400 rem	50% probability of death within 30 days
space shuttle astronaut	25 rem	due to increased cosmic ray exposure
accidental exposure	10 rem	blood changes barely detectable
max. allowed exposure for radiation workers	5 rem over 1 year	no blood changes detectable, negligible increased risk of cancer.
radon exposure (avg. US)	200 mrem = 0.2 rem/yr	probably none
other terrestrial sources	40 mrem/year	probably none
cosmic radiation (sea level)	30 mrem/ year	probably none
single chest x-ray	20 mrem	probably none
nuclear fallout <sup>+</sup>	3 mrem/year	probably none
nuclear power plant leakage	0.01 mrem/year	probably none
total average dose (US citizens)	350 mrem/year	probably none

<sup>+</sup> primarily due to atmospheric testing of nuclear weapons by US and USSR in the 50's and early 60's, prior to the nuclear test-ban treaty which forbid above-ground testing.

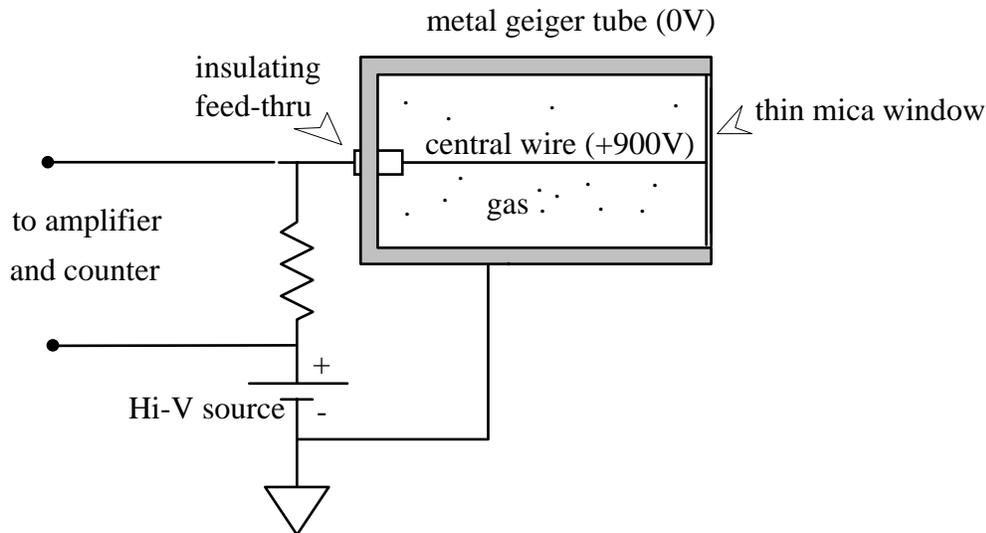
The numbers in the table above generally refer to whole-body exposure. During radiation treatments, cancer patients typically receive 6000 rem, but in a very localized region (the tumor, which is killed!).

### Geiger Counters

A Geiger counter is a simple device for measuring radioactivity. It consists of a metal tube containing a low pressure gas and a wire along its central axis. The wire is held at a high positive voltage ( $\approx 400 - 1000\text{V}$ ). One end of the tube usually has a very thin (and fragile!) window, made of some low-atomic mass material, such as mica or beryllium, through which radiation can easily enter. A single  $\gamma$  or  $\beta$  particle, entering the Geiger tube, can collide with a gas atom and ionize it, stripping it of some electrons. These electrons are accelerated toward the positive central wire by the strong electric field between the grounded outer case and the high-voltage wire. They collide with other atoms, causing further ionization, resulting in avalanche of ionization events and a pulse of current which is detected by external circuitry and makes an

audible chirp in a speaker. In this way, a single high-energy particle can produce a large, brief signal which is easily distinguished from electronic noise.

The mica windows on our Geiger counters, although quite thin, are too thick to allow the passage of  $\alpha$  particles.  $\beta$  particles can pass through the mica window, but not the aluminum sides of the tube.  $\gamma$  particles can enter the tube from all directions: front, back and sides. The detection efficiency of Geiger counters for  $\gamma$  rays is only about 1%, that is, for every 100  $\gamma$ -ray photons which pass through the detector, only one triggers an ionization avalanche. The detection efficiency for  $\beta$  particles is 90%.



### Counting Statistics

Radioactive decay is a random process. Suppose that in a particular experiment,  $N$  counts are recorded in a time  $T$ . Then the counting rate, for that trial, is  $R = N/T$ . If the *average* counting rate is  $\bar{R}$  (the bar means that the average is taken over many trials), then any particular trial will result in a measured  $R$  which will probably be close to, but not exactly equal to  $\bar{R}$ . A very similar situation occurs if you flip a coin many times: the number of heads will be close to, but seldom exactly equal to, the number of tails. In such cases, the counting statistics have a simple rule, called the  $\sqrt{N}$  rule: If the number of counts recorded in a time  $T$  is  $N$ , then the uncertainty in the number of counts is  $\sqrt{N}$ . That is, if you repeat the experiment many times, you will get a gaussian distribution of  $N$ 's, centered on  $\bar{N}$ , the average value of  $N$ , with a standard deviation of  $\sqrt{\bar{N}}$ . If you do the experiment just once, then the best estimate of the count is  $N \pm \sqrt{N}$ , and the best estimate of the count rate is  $R = \frac{N}{T} \pm \frac{\sqrt{N}}{T}$ . The fractional

uncertainty of the count  $\frac{\delta N}{N}$  is equal to the fractional uncertainty of the count rate since

$\frac{\delta R}{R} = \frac{\delta(N/T)}{N/T} = \frac{\delta N}{N}$  so we have  $\frac{\delta R}{R} = \frac{\delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$ . Notice that the fractional uncertainty decreases as  $N$  increases.

When you measure the count rate near a radioactive source, the Geiger counter always measures the *total* rate due to the source and the background. How do you remove the background from the measured rate to get the rate due to the source only, and how does this affect your uncertainty? This question is particularly important if you suspect that you are near a weak source of radioactivity, a source which increases the radioactivity at your location only slightly above background. How do you decide whether the source is actually present?

If a Geiger counter counts for some period  $T$ , the total count  $N_T$  will be the count due to the source  $N_S$  plus the count due to the background  $N_B$ .

$$(1) \quad N_T = N_S + N_B$$

or, written in terms of rates,

$$(2) \quad R_T = R_S + R_B.$$

The background rate  $R_B$  can be determined simply by moving the source far away and re-measuring with the Geiger counter. The rate due to the source alone is

$$(3) \quad R_S = R_T - R_B.$$

To get the uncertainty in  $R_S$ ,  $\delta R_S$ , we use the rule for propagation of errors in addition or subtraction:

$$(3) \quad \delta R_S = \sqrt{(\delta R_T)^2 + (\delta R_B)^2}.$$

**Example:** A Geiger counter is placed near a suspected source of radioactivity and it records 58 counts in 30 sec. The source is removed and the background count is found to be 48 counts in 30 sec. Can we be sure that the source is truly radioactive?

**Answer:**

The total count rate is

$$R_T = \frac{N_T}{T} = \frac{58 \text{ ct}}{0.5 \text{ min}} = 116 \frac{\text{ct}}{\text{min}}, \quad \delta R_T = \frac{\delta N_T}{T} = \frac{\sqrt{N_T}}{T} \cong 15 \frac{\text{ct}}{\text{min}}.$$

The background count rate is

$$R_B = \frac{N_B}{T} = \frac{48 \text{ ct}}{0.5 \text{ min}} = 96 \frac{\text{ct}}{\text{min}}, \quad \delta R_B = \frac{\delta N_B}{T} = 14 \frac{\text{ct}}{\text{min}}.$$

The computed source count rate is

$$R_S = R_T - R_B = 116 - 96 = 20 \frac{\text{ct}}{\text{min}}.$$

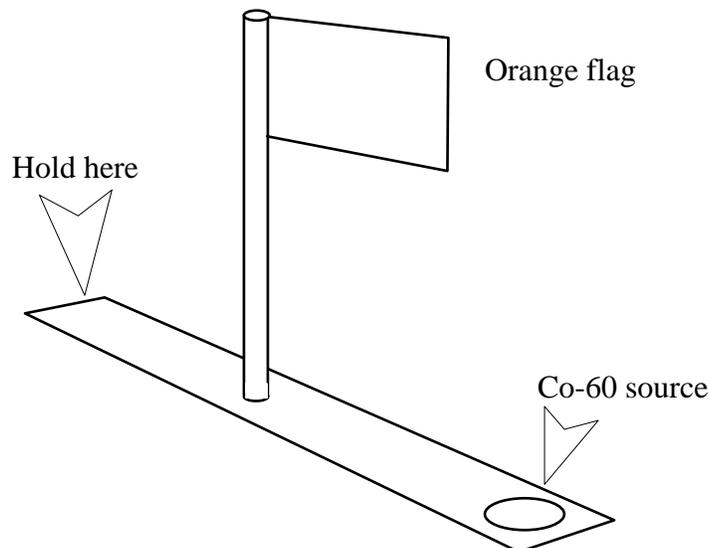
The uncertainty in the source rate is given by

$$\delta R_S = \sqrt{(\delta R_T)^2 + (\delta R_B)^2} = \sqrt{15^2 + 14^2} \cong 21 \frac{\text{ct}}{\text{min}} .$$

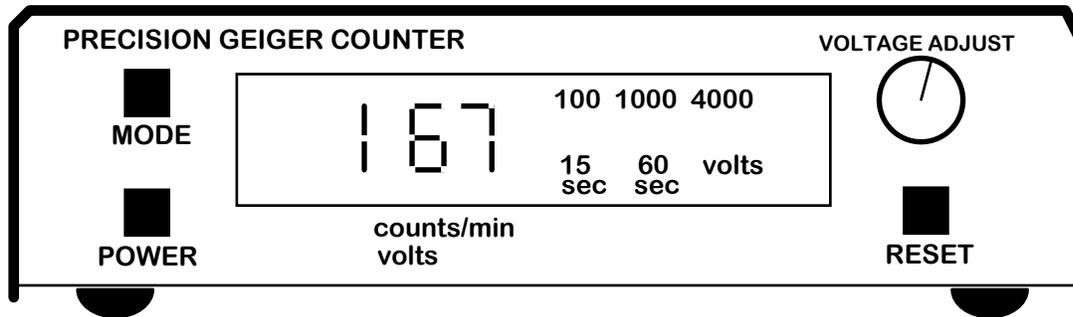
The computed source rate is  $R_S = 20 \pm 21$  ct/min. So, based on the available data, it is quite possible that the increased count observed when the suspected source was present was simply a random fluctuation and not due to increased radioactivity. Longer counting times would be required to resolve the issue.

### Procedure

The radioactive source which you will use in this experiment is a sealed Co-60  $\gamma$ -emitter with an activity of  $1.0 \mu\text{Ci}$ . (“Cobalt-60” means the isotope with 60 nucleons, i.e. number of protons + neutrons = 60.) The half-life of this isotope is 5.3 years and its decay produces  $\gamma$ -rays of two different energies: 1.17 MeV and 1.33 MeV. It is safe to handle; if you carried one in your pocket for several days, your dose would be a few times greater than that due to natural background radiation, but less than a chest x-ray. Nevertheless, you should treat radioactive sources with a healthy respect. Never touch a radioactive source with your hands. Each source in this lab has a long holder.



Before turning on the Geiger counter, make sure that the *Voltage Adjust* knob (on the right) is turned all the way down (CCW). Turn on the counter, and press the MODE button a few times until the *volts* label is illuminated. The display now reads the voltage on the Geiger counter tube. *Slowly*, turn up the voltage until the voltage reads  $900 \pm 10$  V (the exact voltage is not critical). You will begin to hear the counter chirp occasionally as it detects natural background radiation.



By pressing the MODE button, you can cycle the counter through several operating modes labeled 100, 1000, 4000, 15 sec, and 60 sec. In the 100 mode, when the RESET button is pushed, the counter records the next 100 counts and then computes the counts/min. During the counting, the display reads the total counts so far, but at the end of 100 counts, the display is the counts/min. Since the total count is 100, the uncertainty in the count is  $\sqrt{100} = 10$ , and the fractional uncertainty of the count and count rate is

$$\frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{100}} = 10\% .$$

In the 1000 mode, the fractional uncertainty is  $1/\sqrt{1000} \cong 3\%$ . In the 4000 mode, it is  $1/\sqrt{4000} \cong 1.6\%$ .

In the 15 sec mode, when the RESET button is pushed, the counter records counts for the next 15 sec and then computes and displays the counts/min. During the counting, the display shows the remaining time. Since the counts/min was computed based on a recorded counts in 15 sec = 0.25 min, the number N of actual counts is 1/4 of the displayed counts/min. From this, the fractional uncertainty in the count,  $1/\sqrt{N}$ , which is the same as the fractional uncertainty in the rate, counts/min, can be computed. In the 60 sec mode, the counter records for 60 sec = 1 min, so the final displayed counts/min is the same as the count N, and the fractional uncertainty is simply  $1/\sqrt{(\text{displayed counts}/\text{min})}$ .

### Part 1. Measurement of background rate

In this part, your goal is to measure the background count rate, due to naturally occurring radioactivity, with an accuracy of better than 10%. Although we could use the Geiger counter in the 1000 mode and get an uncertainty of  $\frac{1}{\sqrt{1000}} \cong 3\%$ , this would take an inconveniently long time, because the background count rate is rather low. Making a single measurement with the Geiger counter in the 100 mode or the 60 second mode would not take long; unfortunately, that would not provide the necessary accuracy. To get the necessary accuracy, but not take too long, we are going to use Geiger counter in the 60 sec mode, but make 5 trials and combine the results.

Remove all radioactive sources from your position by placing them across the room, behind the lead brick barrier. With the Geiger counter in the 60 sec mode, measure the background count. Repeat for 4 more trials. Using your data for all 5 trials (total time of 5 min), compute the background rate (in counts/min) and its uncertainty. To do this, just imagine that instead of five 1-min counts, you performed a single 5-min count. The total count for the 5 min

period is the sum of the 1-min counts,  $N_{\text{tot}} = \sum_i N_i$  and the total time is  $T = 5$  min. The rate is

$$R = \frac{N_{\text{tot}}}{T} \text{ and the uncertainty is } \delta R = \frac{\sqrt{N_{\text{tot}}}}{T}.$$

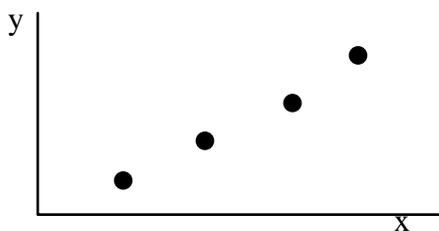
## Part 2. Penetration of $\gamma$ 's through lead sheets

Place the Co-60  $\gamma$  source in the tray under the Geiger counter tube. You should hear the rate on the counter increase dramatically. With the Geiger counter in the 1000 mode, measure the rate due to the source + background. Then compute the rate due to the source only and compute its uncertainty. Remember, in the 1000 mode, the Geiger counts to  $N=1000$ , and then computes the rate  $R$ . You do not know the time  $T$ , so you can't use the formula  $\delta R = \frac{\delta N}{T}$ .

Instead, you must use  $\frac{\delta R}{R} = \frac{\delta N}{N} = \frac{1}{\sqrt{N}}$ .

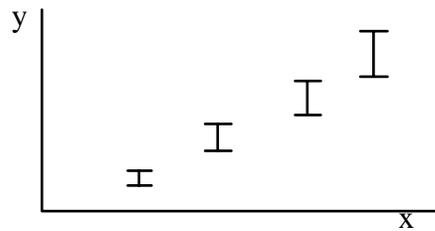
There are 5 lead sheets which can be placed in the slots in the tray between the  $\gamma$ -source and the Geiger tube. (Lead is a fairly toxic material, so either handle the sheets with the big tweezers or be sure to wash your hands at the end of the lab.) This experiment works best if you stack the lead plates starting at the bottom slot, near the source.

With the counter still in the 1000 mode, measure the rate with  $n = 1$  lead sheet in place, then with  $n = 2, 3, 4,$  and 5 sheets. For each  $n$ , compute the rate due to the source only and its uncertainty. Finally, make a graph with error bars of the rate  $R$  vs. number of sheets  $n$ . (Instructions for putting error bars on graphs in Mathcad can be found in the file `errordoc.mcd` in `c:\winmcd` or near the experiment.)



Ordinary graph

(Just an example, NOT a graph of  $R$  vs.  $n$ .)



Same graph with error bars

The flux of  $\gamma$ 's which can penetrate a lead wall decreases exponentially with the thickness of the wall. If  $R_0$  is the rate measured with no lead wall present, then the rate with a lead wall of thickness  $x$  is given by

$$(4) \quad R = R_0 e^{-x/\lambda},$$

where  $\lambda$ , called the penetration depth, is the thickness at which the measured rate has dropped by a factor of  $1/e$ . At a thickness of  $2\lambda$ , the rate is down to  $R_0/e^2$ . At  $3\lambda$ , the rate is down to  $R_0/e^3$ , etc. By taking the natural log of both sides of eq'n(4), we obtain

$$(5) \quad \ln R = -\frac{x}{\lambda} + \ln R_0.$$

Thus, a graph of  $\ln R$  vs.  $x$  should be a straight line with slope  $= -1/\lambda$ .

The thickness of each lead sheet is  $x_0 = 1.57$  mm. If the number of sheets is  $n$ , then the total thickness is  $x = n \cdot x_0$ , and eq'n (5) can be written as

$$(6) \quad \ln R = -n \cdot \frac{x_0}{\lambda} + \ln R_0.$$

Thus a graph of  $\ln(R)$  vs.  $n$  should be a straight line with slope  $m = -x_0/\lambda$ .

Using your measured  $R$ 's, make a graph with error bars of  $\ln(R)$  vs.  $n$ . Determine the slope  $m$  and the intercept  $b$  of the best fit line using the file *linfit.mcd*, which computes the best fit line to any  $x$ - $y$  data. The file *linfit.mcd* is on your harddisk. In Mathcad, open *linfit.mcd*, enter your  $x$ - $y$  data ( $x = n$ ,  $y = \ln(R)$ ) and *linfit* computes  $m$ ,  $b$ ,  $\delta m$ , and  $\delta b$ . You can switch back and forth between *linfit.mcd* and your original file with the WINDOW menu item.

From your computed  $m$  and  $b$ , plot the best fit line on your graph of  $\ln(R)$  vs.  $n$ . From the slope  $m \pm \delta m$ , compute the penetration depth  $\lambda \pm \delta \lambda$ .

### PreLab Questions

1. Using a Geiger counter, a radiation safety technician records 200 counts in 50 seconds. What is the total count rate in counts/min? What is the uncertainty in the count rate in counts/min?
2. **(Counts as two questions)** A Geiger counter measures background radiation and records 950 counts in 10 minutes. Then the Geiger counter is brought near a radioactive source and it records 1430 counts in 10 minutes. Compute  $R_s$ , the rate (in counts/min) due to the source only. Also compute  $\delta R_s$ , the uncertainty in the rate due to the source only.
3. Which kind of radiation ( $\alpha$ ,  $\beta$ , or  $\gamma$ ) is the most penetrating? Which kind is the most effective in doing damage to biological organisms?
4. In 1000 mode, the Geiger counter records a rate  $R_1 = 400$  counts per minute. What is  $\delta R_1$ ? In 100 mode, the Geiger counter records a rate  $R_2 = 400$  counts per minute. What is  $\delta R_2$ ?
5. In this lab, you are to make a graph of  $\ln(R)$  vs.  $n$ . If your measured rate  $R$  has an uncertainty  $\delta R$ , then what is  $\delta(\ln R)$ , the uncertainty in  $\ln R$ ? [Hint: if  $f = f(x)$ , then  $\delta f = \left| \frac{df}{dx} \right| \delta x$ .]
6. In this lab, you use a program called `linfit.mcd` to determine the slope  $m$  of the graph of  $\ln(R)$  vs.  $n$ . The program `linfit.mcd` also gives the uncertainty  $\delta m$ . How do you compute the penetration depth  $\lambda$  and the uncertainty  $\delta \lambda$  from the slope  $m$  and the uncertainty  $\delta m$ ?
7. Suppose, in this experiment, the penetration depth  $\lambda$  in lead is 9.4 mm (which is not far from the truth). How many lead sheets of thickness  $x_0 = 1.57$  mm are needed to reduce the rate to  $(1/e)^2$  of the rate  $R_0$ ? ( $R_0$  is the rate with no lead sheets.)
8. Sketch what your graph of  $\ln(R)$  vs.  $n$  should look like? (No numbers! Just a qualitative sketch). What are the slope and intercept of this graph?
9. Suppose the background rate is measured 5 times with the Geiger counter in 60 sec mode. The 5 readings of the Geiger counter are 50, 57, 49, 55, and 51 (all in cts/min). From these data, compute the best value of the background rate  $R_b$ , and the uncertainty  $\delta R_b$ .